Eur Phys J C **33**, s01, s868–s870 (2004)
Digital Object Identifier (DOI) 10.1140/epjcd/s2004-03-1842-9

EPJ C direct
electronic only

Leptogenesis and low energy observables

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Received: 25 August 2003 / Accepted: 19 September 2003 /
Published Online: 8 April 2004 –

Abstract. Leptogenesis can link the smallness of neutrino masses as implied by neutrino oscillations with the non–vanishing baryon asymmetry of the universe. This connection is provided by the see–saw mechanism. It is interesting to ask if one can relate also the CP violation required for leptogenesis at high energy with the CP violation at low energy as measurable in neutrino oscillation experiments or neutrinoless double beta decay. Though in general this is not possible, various approaches can very well link these phenomena. An Ansatz with minimal input – namely a hierarchical Dirac mass matrix – is presented and its consequences for leptogenesis, neutrino mixing and neutrinoless double beta decay are analyzed.

PACS. 14.60.St 12.60.-i 13.35.Hb 14.60.Pq

1 Introduction

Recent years saw tremendous progress in the understanding of the form of the neutrino mass matrix

$$m_\nu = U \, m_\nu^{\rm diag} \, U^T \; , \qquad (1)$$

where U is the unitary Pontecorvo–Maki–Nagakawa–Sakata (PMNS) lepton mixing matrix. It can be parametrized as

$$U = \left(\begin{array}{ccc} c_1 c_3 & s_1 c_3 & s_3 e^{-i\delta} \\ -s_1 c_2 - c_1 s_2 s_3 e^{i\delta} & c_1 c_2 - s_1 s_2 s_3 e^{i\delta} & s_2 c_3 \\ s_1 s_2 - c_1 c_2 s_3 e^{i\delta} & -c_1 s_2 - s_1 c_2 s_3 e^{i\delta} & c_2 c_3 \end{array} \right) P$$
$$\text{with } P = {\rm diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}) \, , \qquad (2)$$

where $c_i = \cos\theta_i$, $s_i = \cos\theta_i$, $i = 1, 2, 3$ and $m_\nu^{\rm diag}$ is a diagonal matrix containing the neutrino masses. See [1, 2] for details. In (1, 2) it is assumed that neutrinos are Majorana particles, which is a consequence of the most popular and appealing mechanism for the generation of small neutrino masses, the see–saw mechanism [3].

In the see–saw mechanism the neutrino mass matrix is given by

$$m_\nu \simeq -m_D \, M_R^{-1} \, m_D^T \; , \qquad (3)$$

where M_R (m_D) is a Majorana (Dirac) mass matrix. It is assumed that $M_R \gg m_D$. Aside from explaining the smallness of neutrino masses, there are two additional predictions:

1. Neutrinos are Majorana particles
   This immediately opens up the possibility for lepton number violating processes such as neutrinoless double beta decay ($0\nu\beta\beta$). It can of course be tested through experiments searching for $0\nu\beta\beta$.
2. There are additional heavy Majorana neutrinos with a corresponding mass matrix $\simeq M_R$.
   Those heave Majorana neutrinos can be blamed for the non–vanishing baryon asymmetry of the universe (BAU) via the leptogenesis mechanism [4].

Therefore, courtesy of the two predictions, one can hope to test (in principle) part of the see–saw mechanism.

Regarding the BAU, the number that ought to be explained is the ratio of the number of baryons to photons, which is determined as [5]

$$Y_B = \frac{n_B}{n_\gamma} \simeq (6.5\,^{+0.4}_{-0.3}) \cdot 10^{-10} \; . \qquad (4)$$

As shown by Sakharov [6], there are three necessary conditions for the generation of a baryon asymmetry, namely

1. Violation of baryon number
2. Violation of the C and CP symmetry
3. Departure from thermal equilibrium

The Standard Model fails to fulfill the third condition and its CP violation would miss the number (4) by many orders of magnitude [7]. From the various new physics approaches to produce $Y_B \neq 0$, leptogenesis is one of the most attractive ones.

2 Leptogenesis

Leptogenesis fulfills all of Sakharov's three conditions for the generation of non–vanishing Y_B. The requisite CP violating asymmetry is caused by the interference of the tree level contribution and the one–loop corrections in the decay rate of the lightest of the three heavy Majorana neutrinos, $N_1 \to \Phi^- \ell^+$ and $N_1 \to \Phi^+ \ell^-$:

$$\varepsilon_1 = \frac{\Gamma(N_1 \to \Phi^- \ell^+) - \Gamma(N_1 \to \Phi^+ \ell^-)}{\Gamma(N_1 \to \Phi^- \ell^+) + \Gamma(N_1 \to \Phi^+ \ell^-)}$$
$$\simeq \frac{1}{8\pi v^2} \frac{1}{(m_D^\dagger m_D)_{11}} \sum_{j=2,3} \mathrm{Im}(m_D^\dagger m_D)^2_{1j} \, f(M_j^2/M_1^2) \; .$$

Here $v \simeq 174$ GeV is the electroweak symmetry breaking scale. The function f stems from vertex and self–energy contributions [4,8]. For $x \gg 1$, i.e., for hierarchical heavy Majorana neutrinos, one has $f(x) \simeq -\frac{3}{2\sqrt{x}}$. Complex m_D, i.e., CP violation implies non–vanishing ε_1 and therefore an excess in leptons. This lepton asymmetry is – at temperatures between roughly 10^{12} and 10^2 GeV – converted into a baryon asymmetry via $B+L$ violating SM processes called sphalerons [9].

The baryon asymmetry is obtained via[1] (see, e.g., [10])

$$Y_B \sim -10^{-2} \, \varepsilon_1 \, \kappa \sim 10^{-4} \, \varepsilon_1 \; , \qquad (5)$$

where κ is a function of several light and heavy neutrino parameters [10], taking into account in how far the out–of–equilibrium is fulfilled. A detailed analysis of the behavior of κ showed, e.g., that the light neutrino masses have to lie in a mass window between 10^{-3} eV and 0.1 eV [10], which are interestingly just the values implied by neutrino oscillation data and the bounds on neutrino masses from cosmological observations [5,11].

3 Connection to low energy observables?

Since the decay asymmetry ε_1 depends on $m_D^\dagger m_D$ and leptonic CP violating effects originate from $m_\nu \sim m_D m_D^T$, one might expect some interplay between these phenomena. However, it can be shown that *in general* there is no such connection. To see this, it is very useful to consider the following parametrization of the Dirac neutrino mass matrix, valid in the basis in which the charged lepton and Majorana mass matrix M_R are real and diagonal [12]:

$$m_D = i \, U \sqrt{m_\nu^{\mathrm{diag}}} \, R \sqrt{M_R} \; . \qquad (6)$$

In this equation R is a complex orthogonal matrix. The quantity on which the decay asymmetry depends is then given by

$$m_D^\dagger m_D = \sqrt{M_R} \, R^\dagger \, m_\nu^{\mathrm{diag}} \, R \sqrt{M_R} \; . \qquad (7)$$

Note that the PMNS matrix has vanished and therefore the low energy phases responsible for leptonic CP violation (as well as neutrino mixing) have in general nothing to do with leptogenesis. In particular, the possibility of no low energy CP violation but nevertheless successful leptogenesis has been pointed out [13].

[1] In supersymmetric versions of the leptogenesis mechanism one obtains roughly the same formula.

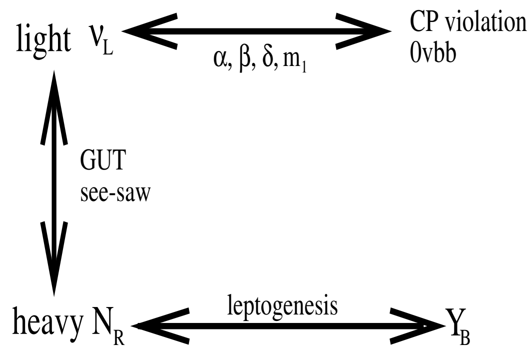

Fig. 1. Connection between low energy lepton number and CP violation with the baryon asymmetry Y_B via the leptogenesis mechanism. Without the *left vertical arrow* there is none

Another way to see this is to note that the total number of parameters in the see–saw model is 18, which may be decomposed as 12 real ones and 6 phases, see e.g. [14]. The amount of low energy parameters in m_ν is exactly half this number[2], decomposable as 6 real parameters and 3 phases in (1, 2). Thus, integrating out the heavy Majorana neutrinos via the see–saw formula (3) leaves us short with a large part of the parameters of the model and spoils any straightforward connection between the low and high energy sector. A symmetry relating low and high energy matrices is therefore required to draw any link [15], see Fig. 1. A given model can thus have very well a connection, and this question has been studied in many approaches [16].

Another possibility to connect leptogenesis with low energy observables exists in supersymmetric frameworks. Assuming universality of all mass matrices at the GUT scale M_X, renormalization group running leads to non–vanishing off–diagonal entries in the slepton mass matrix. These terms are responsible for charged lepton decays as $\ell_i \to \ell_j + \gamma$, $\ell_i(\ell_j) = \tau, \mu, e$ for $i(j) = 3, 2, 1$ and were shown to be proportional to $(m_D m_D^\dagger)_{ij}$ [17]. Taking into account even electric dipole moments of the charged leptons and T asymmetries of $\mu \to 3e$ decays it is – in principle and in a very model dependent way – possible to fully reconstruct the see–saw model.

4 Leptogenesis with hierarchical mass matrices

We shall now present an Ansatz for leptogenesis and neutrino mixing [14]. The only input we shall make will be that the Dirac mass matrix is hierarchical and connected to the known quark or charged lepton masses. Also, the heavy Majorana neutrinos shall display a hierarchy. Our goal will be to have some connection between leptogenesis and low energy observables such as $0\nu\beta\beta$ and to identify under which circumstances this happens. A hierarchical

[2] This happens in general in see–saw models with the same number of light and heavy Majorana neutrinos [14].

Dirac mass matrix can most comfortably be described in the so–called bi–unitary parametrization

$$m_D = U_L^\dagger \, m_D^{\rm diag} \, U_R \; , \qquad (8)$$

where U_L (U_R) is a unitary 3×3 matrix and $m_D^{\rm diag}$ is a diagonal real matrix with entries m_{Di}. The matrices of interest then read

$$\begin{array}{ll} \text{mass matrix: } m_\nu & = -U_L^\dagger \, m_D^{\rm diag} \, U_R \, M_R^{-1} \, U_R^T \, m_D^{\rm diag} \, U_L^\ast \\ \text{leptogenesis: } m_D^\dagger m_D & = U_R^\dagger \, (m_D^{\rm diag})^2 \, U_R \\ \text{LFV decays: } m_D m_D^\dagger & = U_L^\dagger \, (m_D^{\rm diag})^2 \, U_L \end{array} .$$

One can parametrize $U_L^\dagger$ (U_R) in analogy to the PMNS matrix in (2). Assuming $m_{D3} \gg m_{D2} \gg m_{D1}$ and taking the mixing angles in U_L (U_R) as $\theta_{1L(R)} \sim 0.1$, $\theta_{2L(R)} \sim 10^{-2}$ and $\theta_{3L(R)} \sim 10^{-3}$ leads to the desired hierarchical Dirac mass matrix.

The first thing one can do within this Ansatz is to calculate the ratio of the branching ratios of the charged lepton decays, one finds

$$BR(\tau \to \mu + \gamma) \sim 10^2 \, BR(\tau \to e + \gamma) \sim 10^6 \, BR(\mu \to e + \gamma).$$

In order to reproduce the observed neutrino phenomenology in this model, it is required that one of the heavy Majorana neutrinos is much heavier than the other two, see [14] for details[3].

Thus, assuming $M_3 \gg 10^3 \, M_{2,1}$ and $m_{D1} \sim 0$ we can estimate the decay asymmetry as

$$\varepsilon_1 \simeq -10^{-9} \left(\frac{m_{D3}}{\rm GeV}\right)^2 \sin 2\alpha_R \, \frac{M_1}{M_2} \qquad (9)$$

where α_R, β_R and δ_R are the phases in U_R. In order to achieve the required decay asymmetry of $\varepsilon_1 \sim 10^{-6}$ one sees from (9) that $m_{D3} \sim 10^2$ GeV is required, i.e., m_D is connected to the up–quark sector. Furthermore, a rather mild hierarchy between the remaining two Majorana neutrinos $M_2/M_1 \sim 10$ is needed. Using (3), we can calculate the ee entry of m_ν, which will be probed in neutrinoless double beta decay. It reads – again for $M_3 \gg 10^3 \, M_{2,1}$ and $m_{D1} \sim 0$:

$$\langle m \rangle \simeq m_{D2}^2 \, s_{1L}^2 \left(\frac{s_{1R}^2}{M_1} + \frac{e^{2i\alpha_R}}{M_2} \right) \; . \qquad (10)$$

With the indicated conditions one sees from (9, 10) that only the term proportional to $\sin 2\alpha_R$ contributes to Y_B and – since α_R also appears in $\langle m \rangle$ – there is a direct correlation between the rate of $0\nu\beta\beta$ and the baryon asymmetry of the Universe. For a mild hierarchy between the masses M_i, the unknown phases in U_L and U_R spoil any simple connection between $\langle m \rangle$ and Y_B.

One can also analyze the CP violating effects in oscillation experiments. In the given framework, however, one finds that all 6 available phases contribute comparably to the relevant quantities. Thus, no low energy CP violation would be very fine–tuned. Nevertheless, leptogenesis and CP violation "decouple", see [14] for details.

[3] In general, non–vanishing U_{e3} close to its current limit is predicted, see [14].

5 Summary

The BAU and neutrino masses cannot be explained by the Standard Model. The see–saw model explains the smallness of neutrino masses and predicts the Majorana nature of neutrinos as well as the presence of heavy Majorana neutrinos. The latter can via the leptogenesis mechanism explain the BAU. Connecting the required high energy CP violation with the low energy leptonic CP violation would allow to partly test the Sakharov conditions. In general, this turns out to be not possible. However, many models allow to draw such a link. A simple and consistent Ansatz is presented which only assumes a hierarchical Dirac mass matrix and relies on the decoupling of the heaviest Majorana neutrino. A characteristic ratio of lepton flavor violating charged lepton decays is obtained. CP violation in oscillation experiments decouples from Y_B, whereas a connection between neutrinoless double beta decay and the BAU is found. The naive expectation that due to the required lepton number violation there should be some connection between neutrinoless double beta decay and the baryon asymmetry is thus met.

Acknowledgements. I wish to thank S. Pascoli and S.T. Petcov for a fruitful collaboration. This work was supported in part by the EC network HPRN-CT-2000-00152.

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